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THE STUDY OF CERTAIN ASPECTS OF PROBABILITY WITH APPLICATIONS IN COMMUNICATION THEORY

INTERIM REPORT

Grant AFOSR-81-0047

November 28, 1984

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Introduction

This interim report constitutes a summary of research performed under Grant AFOSR-81-0047 during the year beginning October 1, 1983. First we present a list of the personnel involved in the research effort. In the next section we present a summary of the research results that have been achieved. Then in the following section we briefly comment upon the research in progress. This is followed by a list of publications supported during this grant year.

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Research Assistants

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Fu-Sheng Lu (received his Ph.D. in May 1984. His dissertation was entitled "Efficient Methods for the Design of Scalar Quantizers.")

John M. Morrison (presently a Ph.D. candidate in the Department of Mathematics)

Senior Secretary

Joan A. Van Cleave

Summary of Results

In this section we will present a brief summary of our research results published after October 1, 1983. The reference numbers in this section are keyed to the publication list in the last section. Most of our research results are in the areas of quantization theory and detection theory. We will begin our summary by discussing our results in quantization theory; this will be followed by a presentation of our results in detection theory; and finally, we will mention our results in other areas.

Quantization is the process by which data is reduced to a simpler, more coarse representation which is more compatible with digital processing. Loosely speaking, quantization is at the heart of analog to digital conversion. It is an area which has increased in importance in the last few years due to the burgeoning advances in digital technology. The typical goal of quantization is to reduce data to a simpler representation without causing much distortion; that is, the output of a quantizer should be close to the input, with some appropriate measure of distance. An N-level k-dimensional vector quantizer is a mapping $Q: \mathbb{R}^k \to \mathbb{R}^k$ which assigns the input vector x to an output vector Q(x) chosen from a set of N vectors $\{y_i: y_i \in \mathbb{R}^k, i=1,\ldots,N\}$. Generally, the quantizer input is modelled as a random vector X described by a k-variate distribution F. A measure of quantizer performance is the distortion function

$$D(Q,F) = \int d(x,Q(x)) dF(x), \qquad (1)$$

where d: $\mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}$ is an appropriately chosen cost function. An optimal N-level quantizer Q for the random vector X is one that minimizes (1) over the class of all N-level quantizers.

There had apparently been a long-standing belief among researchers in

quantization theory that optimal quantizers always exist. This existence is important from the viewpoint of numerical design algorithms and in studying convergence properties of sequences of quantizers; also, several results in quantization theory are hypothesized upon the existence of optimal quantizers. Several of our earlier efforts were concerned with establishing conditions guaranteeing the existence of optimal quantizers. For the case of difference-based distortion functions, i.e. d(x,y) = C(||x-y||), we completely settled the existence question; in [16] we presented necessary and sufficient conditions for an optimal quantizer to exist. In [15] our interest was primarily concerned with convergence properties of sequences of quantizers; however, as a side result, we did establish a condition guaranteeing existence of optimum quantizers for non-difference based distortion functions d(x,y). This result provided a counterexample to a speculation of Gray, Kieffer, and Linde (Information and Control, May 1980).

We have also been active in establishing convergence properties of sequences of quantizers. These convergence results are important from the viewpoint of numerical design algorithms, and they yield considerable insight into the limiting behavior of sequences of quantizers. Suppose that a sequence of probability measures P_n converges weakly to a probability measure P_n . Let Q_n be an optimal N-level quantizer for P_n . Does the distortion associated with the quantizer Q_n and the measure P_n converge to the optimal distortion for quantizing P with N-levels? Does the sequence of optimal quantizers for the P_n 's converge to a quantizer Q; and if so, is Q optimal for P? Several of our convergence results have been focused about these two questions. In [3] we considered difference based cost functions, e.g. d(x-y), and we established results sufficient for affirmative

answers to the above questions. Then in [15] we established conditions sufficient for affirmative answers for non-difference based cost functions, e.g. d(x,y). In each of the above works we also considered the above two questions where P_n represented the empirical measure based on n iid samples drawn from the measure P, and we established conditions sufficient for almost sure convergence in the above two questions. In all of the above convergence results we chose to put conditions on the cost function rather than the distribution; the cost function is easier to control than the (frequently not exactly known) underlying distribution.

These convergence results for sequences of quantizers are fairly general and they form powerful tools for the study of quantization. For example, one of the more practical problems associated with quantizers is the problem of how to construct them. Most of the algorithms for quantizer design involve successively improving a suboptimal quantizer, with the procedure hopefully converging to an optimal quantizer. The above results in [3] and [15] directly address this situation. For example, one of the currently most popular design algorithms for vector quantization is the so-called "Linde-Buzo-Gray algorithm" (IEEE Transactions on Communications, January 1980). As a by-product of one of our results in [3], we established convergence of this algorithm for r-th power distortion measures, i.e. $d(x,y) = ||x-y||^r$. This is the first rigorous convergence result for this algorithm in a reasonably general context. In [13] we used the results in [3] to investigate convergence properties of quantizer design via successive improvement upon suboptimal quantizers. If the input distribution F were not known, we might form an estimate \hat{F}_n based on n observations of the input signal. As n becomes large, we might expect a reasonable estimate to converge to the true distribution F. Intuitively, then, an optimal

quantizer designed for \hat{F}_n , and the resulting distortion, should closely approximate those of an optimal quantizer for F. In [13] we established properties of an estimator \hat{F}_n so that the above reasoning would be valid.

Another aspect of our work on quantization was concerned with practical, numerically-oriented design techniques for scalar quantizers. Although the advantages of vector, or block, quantization are well known, scalar quantizers are nevertheless in widespread use. In spite of numerous elegant results in quantization theory, the actual practical numerical design of scalar quantizers is still a challenging problem. In [8] we presented a simple and straightforward technique for constructing minimum mean squared error symmetric uniform scalar quantizers for some common distributions on the data.

In the context of scalar minimum mean squared error quantization, one of the most popular design techniques is the Lloyd-Max algorithm (IRE Transactions on Information Theory, March 1960 and IEEE Transactions on Information Theory, March 1982). Unfortunately, two potential problems arise with the Lloyd-Max scheme. The first problem is how to get a good initial guess for starting the iterative scheme, and the second problem is how to intelligently update the algorithm. Both of these problems were addressed in [1] and [18] for some common distributions on the data. Our modifications of the Lloyd-Max algorithm resulted in a very fast design algorithm for scalar minimum mean squared error quantization. For example, we designed a 64-level quantizer for a Gaussian distribution with a high degree of accuracy (the terminating condition for the Lloyd-Max algorithm was set at 10^{-8}) in 0.136 seconds of computer time on a CDC Cyber 170/750.

One of the non-mean-squared error criteria that frequently appears in the literature is the criterion of mean absolute error. In [11] we

presented an efficient method for the design of scalar minimum mean absolute error quantizers. Our method was based upon a modification of the Lloyd-Max algorithm mentioned above. As an example, we designed a 64-level minimum mean absolute error quantizer for a standard normal random variable. Also in [11] we gave a closed-form solution for the minimum mean absolute error quantizer for a Laplace random variable. This closed-form solution stands in marked contrast to the laborious numerical procedures often encountered in quantizer design problems.

A popular way of realizing a scalar quantizer is via a method known as companding. A companding system consists of an invertible function $G\colon \mathbb{R} \to [0,1]$ followed by a uniform N-level quantizer on [0,1], followed by the inverse function $G^{-1}(\cdot)$. Any arbitrary N level scalar quantizer can be realized via a companding system. This technique leads to a closed form solution; however, it is asymptotic in nature. In some cases the accuracy of the companding method has been overrated. In [7] we presented a simple modification for improving the accuracy of the companding scheme. For the generalized Gaussian density, $f(x) = A \exp[-c|x|^p]$, this modification resulted in a straightforward formula for constructing a better compressor function G.

Another research area in which we have recently obtained results is the area of signal detection. The detection problem is modeled as a test between two statistical hypotheses; we assume that under the null hypothesis noise alone is being observed, and under the alternate hypothesis a signal plus noise is being observed. We considered discrete time detection. Where we assumed that the observation is indexed by a subset of the integers, e.g. x_1, x_2, \ldots, x_n .

In the case of discrete time detection where the noise and the signal

are stationary and the samples are independent, it is well known that the Neyman-Pearson test has a test statistic which can be expressed as

$$\sum_{i=1}^{n} g(X_i)$$

where X_i , i=1,...,n, represent the observations, and $g(\cdot)$ is an appropriately chosen function. In earlier work we had considered the problem of constraining the test statistic to be of the above form and letting the noise samples be "slightly" dependent. We then tried to choose the function $g(\cdot)$ to best account for the dependency structure of the noise, in the sense of the asymptotic relative efficiency (or Pitman efficiency) with respect to any other choice for $g(\cdot)$. In [4] we investigated the problem of how to choose $g(\cdot)$ when both the signal and the noise were modelled as ϕ -mixing random processes, where we also allowed the noise to be dependent on the signal over a finite window, such as signal dependent noise induced through reverberation effects. In [5] we considered the problem of approximating an optimal $g(\cdot)$ by a sequence of Borel measurable functions $\{g_i(\cdot)\}$. We compared the performance resulting from the approximate nonlinearities to the optimal performance, and we showed that the loss in performance can be made arbitrarily small by making $g_i(\cdot)$ appropriately close to $g(\cdot)$. We allowed a strong mixing dependency structure for the (random) signal and the noise, and we considered as examples specific forms, e.g. quantizers, polynomials, for the $g_i(\cdot)$. In [6] and [19] we continued part of this investigation. Here we were concerned specifically with approximating the nonlinearity $g(\cdot)$; and our interest was in establishing a lower bound on the performance, where the lower bound was a function of the L_2 distance between the optimal $g(\cdot)$ and the actual nonlinearity of interest. Notice that for several reasons one might not use the optimal $g(\cdot)$; for example,

numerical approximations may be employed in solving for $g(\cdot)$, some of the statistical information necessary for determining $g(\cdot)$ may only be approximated, or perhaps one introduces another nonlinearity in an attempt to lend robustness properties to the detection scheme. Our results in [6] and [19] directly address the question of how the asymptotic performance is degraded by perturbations in $g(\cdot)$.

The relative efficiency between two detectors is a ratio of the amount of data required by one detector, relative to another, to attain a prescribed level of performance. Although this concept is of fundamental importance in the theory of signal detection, it has been successfully investigated in only very few special cases. As an approximation to the relative efficiency, engineers have frequently employed the asymptotic relative efficiency (ARE), the limiting value of the relative efficiency (under suitable regularity conditions) as the sample sizes required by the detectors approach infinity. The ARE was introduced in the statistical literature, where it is known as the Pitman efficiency. Usually it can be determined in a fairly straightforward fashion, and this is due principally to an appeal to the central limit theorem. The ARE is a limiting result; and in any practical engineering situation, only a finite number of samples can be taken in the context of discrete time detection. Thus it might not always be appropriate to approximate the relative efficiency with the ARE. In [10] we considered the discrete time detection of a known time varying signal in additive noise, where the noise sequence is assumed to be a sequence of iid random variables; and we studied the relative efficiency of the sign detector, a popular nonparametric detector, and the correlation detector, which is Neyman-Pearson optimal in the case when the noise is Gaussian. In this work [10] we presented results illustrating the convergence of relative

efficiencies for both Gaussian noise and Laplace noise. Some examples were given where the relative efficiencies did not quickly converge to the ARE. In this work [10] we also presented bounds on the relative efficiency in the case where the (deterministic) signal was unknown; for example, it might only be known that at the i-th sample, $\bar{s}_i - \epsilon \le s_i \le \bar{s}_i + \epsilon$, where s_i represents the signal, and \bar{s}_i and ϵ are known.

Kassam and Thomas (IEEE Transactions on Information Theory, July 1975) considered the discrete time detection of a constant signal in m-dependent noise. This scheme consisted of summing the first n samples, skipping (i.e. throwing away) the next m, summing the next n, skipping the next m, etc. They then applied the classical sign detector to the sequence of sums, and they concluded that, asymptotically, n should be chosen as large as possible to maximize performance. They then concluded that this method could be extended to noise sequences that were strong mixing, and that results under an m-dependent assumption yielded very close approximations. In [17] we presented a rigorous analysis of this conjectural conclusion, and we showed that a considerably more careful analysis was necessary for the case of strong mixing noise. We showed how such a nonparametric detector may be designed. We established an upper bound on the asymptotic performance and we specified the form of a detector which achieves this upper bound. [17] we also considered the design of the detector under a finite sample (i.e. non-asymptotic) criterion, and we showed that there can be a marked difference in the detector designs resulting from the two criteria (i.e. asymptotic and non-asymptotic).

Consider detecting a deterministic time varying signal in additive noise based on a fixed (finite) number of observations. If the noise process is mutually independent, then the solution of the problem is easily

formulated in terms of the Neyman-Pearson criterion in which the detection probability is maximized for a constrained false alarm probability. The resultant detector is then implemented by comparing the output of a transformation of the data to a threshold, the transformation being obtainable from the univariate noise distributions. However, with today's high sampling rates, an assumption of independent samples is becoming increasingly inappropriate. Although the Neyman-Pearson criterion can still be applied in theory, the presence of dependency greatly compromises its application. Lack of knowledge of the higher order noise distributions results in the inability to specify completely the required transformation (the likelihood ratio). We therefore have a situation in which the problem is tractable under an independence assumption but it should most properly be approached under the dependence assumption. Often in the past, whatever dependency has been present has been ignored in order to obtain tractable results. This has led to variations in the nominal values of the detection probability and the false alarm probability because of the residual dependency. If the dependency was "weak", then one would hope that these variations would be acceptably small. In [2] and [14] we investigated quantitative conditions which allowed determining when the dependency can be ignored, and we presented a result which allowed bounding the variations in the detection probability and the false alarm probability induced by ignoring the dependency.

Consider once again the discrete time detection of a signal in additive noise. Under a variety of fidelity criteria, an optimal detector consists of mapping the data into the real numbers via the likelihood ratio and then comparing the result to an appropriate threshold (determined by the fidelity criterion). Clearly, the likelihood ratio represents the

actual "processing" of the data. Assume that the noise distribution is changed from its nominal model. When does the resulting likelihood ratio (i.e. the data processor) change? In [12] we considered this situation and we completely characterized the situation where the noise distribution can change but the likelihood ratio remains unchanged. In particular, we produced examples where the noise distribution can change dramatically, but the likelihood ratio remains the same.

In [9] we investigated an existing method (Delp and Mitchell, IEEE Transactions on Communications, September 1979) for image compression known as block truncation coding. The basic block truncation coding approach employs a two level quantizer whose output levels are obtained through matching the first two sample moments of the data before and after quantization. We generalized this basic block truncation coding approach by using two level quantizers which preserve higher order moments. This generalization offered the potential for improved performance. Some examples were given to illustrate the improvement in image quality.

Finally, in [20] we pointed out that even for bounded random variables the conditional expectation does not always yield a minimum mean squared error estimate. That is, we constructed two bounded random variables X and Y and a function $f: \mathbb{R} \to \mathbb{R}$ such that Y = f(X) pointwise on the underlying probability space but $E\{(Y-E\{Y|X\})^2\} > 10^{10}$.

Research in Progress

Our research is progressing very well in several directions. In this section we will briefly describe the problems we are currently investigating.

The newest research direction we are pursuing and the one in which most of our effort is currently being expended deals with several aspects of conditional expectations. Naturally, this is closely aligned with mean squared error estimation. For example, let Y denote a second order random variable of interest, and let X_1,\ldots,X_k denote our data. One might decide to estimate Y by using $\hat{Y} = E\{Y|X_1,\ldots,X_k\}$. However, in a practical situation, the data is better modeled as $Q_1(X_1),Q_2(X_2),\ldots,Q_k(X_k)$, the result of an analog to digital conversion of the observations. This analog to digital conversion would be the result of the digitization of the observations; for example, they might be stored in a digital computer. Thus, perhaps we should use as our estimate of Y the quantity $\hat{Y} = E\{Y|Q_1(X_1),\ldots,Q_k(X_k)\}$. How does $E\{(Y-\hat{Y})^2\}$ compare with $E\{(Y-\hat{Y})^2\}$? How should the quantizers $\{Q_{\hat{I}}\}$ be designed to make $E\{(Y-\hat{Y})^2\}$ close to $E\{(Y-\hat{Y})^2\}$?

Another aspect of our investigations deals with the continuity of σ -algebras generated by random processes. Let X(t) denote a random process, and let $F_t = \sigma\{X(s), s \le t\}$. Define

$$F_{t+} = \bigcap_{s>t} F_s$$

and

$$F_{t-} = \sigma \{ \bigcup_{s < t} F_s \}.$$

We say that the flow $\{F_t\}$ is continuous at t if $F_{t-} = F_t = F_{t+}$. In numerous popular works on estimation theory (e.g. Gihman and Skorohod,

The Theory of Stochastic Processes, Vols. I. II. III and Liptser and Shiryayev, Statistics of Random Processes I and II), it is simply assumed that the flow of σ -algebras is continuous, and this assumption plays a fundamental role in many of the results. How restrictive is this assumption? We are currently investigating properties of X(t) that are consistent with the continuity of F_+ . Our present results indicate that there is little if any relation between the sample path regularity of X(t) and the continuity of F_{+} . For example, we can exhibit random processes with real analytic sample functions and discontinuous o-algebras, and we can exhibit random processes with non-Lebesgue measurable sample functions and continuous σ -algebras. As implied earlier, the results of this investigation are pertinent to the applicability of the results in several popular texts. Also, these results are fundamental to relating estimates based on observing a random process over an interval to estimates based on observing a random process at only a finite set of times. For example, how does

$$E{Y|X(s), se[a,b]}$$

compare to

$$E{Y|X(s_i), i=1,...,n},$$

where the $s_i \in [a,b]$? Can we make them close in some sense? How should the observation times s_i be chosen?

One of the more practical problems we are investigating is data reduction for image processing. Consider an image composed of pixels taking on one of several gray levels. For example, if there are 2^b gray levels, then each pixel can be represented by using b bits, and each image would therefore be representable by a certain number of bits. We are presently investigating a method for reducing the number of bits used to represent

an image without altering the image very much. Our results in this area are still in an embryonic stage. We hope to characterize a class of images and a method of data reduction so that the data can be reduced by a factor, say k:1, and at the same time the image will undergo only negligible alteration.

Our current work in the theory of signal detection is moving away from asymptotic results and more toward detection based on a finite number of observations. Two main directions in our investigation of signal detection are concerned with properties of the relative efficiency between detectors and with consequences of robustness in detection schemes. Engineers have often used the asymptotic relative efficiency between two detectors as a way of comparing the detectors. However, in a practical situation, the quantity of concern is actually the relative efficiency (based on a finite number of samples). As mentioned in the previous section, we have already achieved some preliminary results in this area. Another aspect of signal detection that we are currently investigating is concerned with the concept of robust signal detection. A saddle point approach to robust hypothesis testing was established in the 1960's by Peter Huber. In the last few years several investigators in signal processing have applied these results to some situations in signal detection. However, there appears to be an inadequate degree of understanding concerning the performance of these robust detection schemes in particular situations. For example, consider a simple hypothesis test using a nominal distribution. Now consider testing composite hypotheses by letting the underlying noise distribution be allowed to vary from the nominal distribution within appropriately defined neighborhoods (e.g. Prohorov distance, Kolmorgorov distance, Levy distance, etc.), and consider a robust detector designed

for this second situation. Naturally, this has the pleasing attribute of being robust, but the question remains as to how the performance is affected by using the robust detector. For example, assume that the robust detector is used for the nominal distribution. How much worse will the performance be than if the Neymann-Pearson detector had been used? Our present investigations are addressing this matter, and we have found some cases where the robust detector for the nominal distribution gave a detection probability less than half of that given by a Neymann-Pearson detector (where both detectors had the same false alarm probability).

The above summary describes our ongoing research. In the near future we hope to focus more on data processing schemes designed under imperfect or erroneous assumptions.

List of Publications

- (1) F.S. Lu and G.L. Wise, "A Further Investigation of the Lloyd-Max Algorithm for Quantizer Design," PROCEEDINGS OF THE TWENTY-FIRST ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 5-7, 1983, pp. 481-490.
- (2) D.R. Halverson and G.L. Wise, "Signal Detection When Dependence Is Neglected," PROCEEDINGS OF THE TWENTY-FIRST ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 5-7, 1983, pp. 622-630.
- (3) E.F. Abaya and G.L. Wise, "Convergence of Vector Quantizers with Applications to Optimal Quantization," SIAM JOURNAL ON APPLIED MATHEMATICS, Vol. 44, pp. 183-189, February 1984.
- (4) D.R. Halverson and G.L. Wise, "Asymptotic Memoryless Discrete Time Detection of (phi)-Mixing Signals in (phi)-Mixing Noise," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-30, pp. 415-417, March 1984.
- (5) D.R. Halverson and G.L. Wise, "Approximately Optimal Memoryless Detection of Random Signals in Dependent Noise," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-30, pp. 420-424, March 1984.
- (6) D.R. Halverson and G.L. Wise, "On the Approximation of Nonlinearities for Memoryless Detection," PROCEEDINGS OF THE 1984 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Princeton, New Jersey, March 14-16, 1984, pp. 302-305.
- (7) F.-S. Lu and G.L. Wise, "A Modified Companding Method for Quantization," PROCEEDINGS OF THE 1984 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Princeton, New Jersey, March 14-16, 1984, pp. 577-580.
- (8) F.-S. Lu and G.L. Wise, "A Simple Approximation for Minimum Mean Squared Error Symmetric Uniform Quantizers," IEEE TRANSACTIONS ON COMMUNICATIONS, Vol. COM-32, pp. 470-474, April 1984.
- (9) D.R. Halverson, N.C. Griswold, and G.L. Wise, "A Generalized Block Truncation Coding Algorithm for Image Compression," IEEE TRANSACTIONS ON ACOUSTICS, SPEECH, AND SIGNAL PROCESSING, Vol. ASSP-32, pp. 664-668, June 1984.
- (10) Y.-C. Liu and G.L. Wise, "On the Relative Efficiency Between the Correlation Detector and the Sign Detector," to appear in PROCEEDINGS OF THE 27TH MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, Morgantown, West Virginia, June 11-12, 1984.

(11) F.-S. Lu and G.L. Wise, "An Efficient Method for the Design of Minimum Mean Absolute Error Quantizers," to appear in PROCEEDINGS OF THE 27TH MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, Morgantown, West Virginia, June 11-12, 1984.

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